MHD Natural Convection Flow of Fluid with Variable Viscosityfrom a Porous Vertical Plate

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Abstract

The behaviour of magnetohydrodynamic (MHD) natural convection flow from a porous vertical plate is studied with respect to variable viscosity. To model the flow, the governing boundary layer equations are first transformed into a non dimensional form. The resulting non linear system of partial differential equations is then solved numerically using finite difference method together with Keller-Box scheme. The study shows that the variation of viscosity influences the surface shear stress (skin friction coefficient) and the rate of heat transfer (local Nusselt number). It has an effect on velocity as well as temperature profiles. Studied has been performed with a selection of parameters set consisting of magnetohydrodynamic parameter M, viscosity variation parameter γ, Prandtl number Pr. Results are shown graphically and tabular form and the comparison agrees well with a published paper.

Keywords

Porous Plate; Magnetohydrodynamic; Natural Convection; Variable Viscosity

Nomenclature

a_r	Rossel and mean absorption co-efficient
C_f	Local skin friction coefficient
C_p	Specific heat at constant pressure
f	Dimensionless stream function
g	Acceleration due to gravity
k	Thermal conductivity
Nu_x	Local Nusselt number
Pr	Prandtl number
Q	Heat generation parameter
q_w	Heat flux at the surface
q_c	Conduction heat flux
q_r	Radiation heat flux
R_d	Radiation parameter
T	Temperature of the fluid in the boundary layer
T_∞	Temperature of the ambient fluid
T_w	Temperature at the surface
(u, v)	Dimensionless velocity components along the (x, y)axes
V	Wall suction velocity $T_w - T_\infty$
(x, y)	Axis in the direction along and normal to the surface respectively

Greek symbols

α	Equal to $\frac{4}{3}R_d$
β	Coefficient of thermal expansion
Δ	Equal to $\theta_w - 1$
ΔT	Equal to
η	Similarity variable
θ	Dimensionless temperature function
θ_w	Surface temperature parameter
μ	Viscosity of the fluid
ν	Kinematic viscosity
ξ	Similarity variable
ρ	Density of the fluid
σ	Stephman-Boltzman constant
σ_s	Scattering co-efficient
μ_f	Absolute Viscosity at the film temperature
τ	Coefficient of skin friction
τ_w	Shearing stress
ψ	Non-dimensional stream function

Subscripts

w	wall conditions
∞	Ambient temperature

Introduction

From technical point of view, the electrically conducting fluid flow in presence of magnetic field is

significant. For the last couple of decade, the electrically conducting fluid flow problems have received much attention from many researchers. The electrically conducting fluid over an surface that flows and transfers heat needs to study for the sake of resolving many mechanical, hydro dynamical engineering and astrophysical problems including solar structure, especially in the outer layers, the solar wind bathing and interstellar magnetic fields.

The main application of magneto hydrodynamic fluid flow is in the astrophysical and geophysical field. However, as material walls are destroyed during creation and containment of hot plasmas during generating fusion power by electromagnetic forces, to resolve the problem, MHD fluid flow is applied. MHD free convection flow with variable viscosity from porous vertical plate has attracted attention not only for its fundamental aspects but also for its significance in the contexts of space technology involving high temperature.

The interaction of the magnetic field and the moving electric charge carried by the flowing fluid induces a force, which tends to oppose the fluid motion. The magnetic force is proportional to the magnitude of the longitudinal velocity of fluid. Near the leading edge of the surface, the velocity is very small that the magnetic force acts in the opposite direction and is also minuscule. Consequently, the influence of the magnetic field on the boundary layer is exerted only through induced forces within the boundary layer itself.

Merkin(1972) concluded free convection with blowing and suction. Lin & Yu(1988) studied free convection on a horizontal plate with blowing and suction. Hossain et al. [Hossain, Khanafer and Vafai (2001)] studied the effect of radiation on free convection flow with variable viscosity from a porous vertical plate. Hossain et al. [Hossain, Munir and Rees (2000)]studied flow of viscous incompressible fluid with temperature dependent viscosity and thermal conductivity past a permeable wedge with variable heat flux. Hossain &Takhar (1996) studied radiation effect on mixed convection along a vertical plate with uniform surface temperature. Molla et al. [Molla, Hossain and Yao (2004)]examined natural convection flow along a vertical wavy surface atuniform surface temperature in the presence of heat generation/ absorption. Akhter (2007) studied the effect of radiations on free convection flow on sphere with isothermal surface and uniform heat flux. Ali (2007)studied the effect of radiation on free convection flow on sphere with heat generation. Hossain et al. [Hossain, Alim and Rees (1999)] investigated the effect of radiation on free convection flow from a porous vertical plate. They [Hossain, Alim and Rees (1999)] performed a full numerical solution and found an increment in radiation parameter $R_d$which makes the boundary layer thin and found an increment in surface temperature parameter which causes the boundary layer to be thick. Molla et al. [Molla, Hossain and Taher (2006)] studied the magnetohydrodynamic natural convection flow on a sphere with uniform heat flux in presence of heat generation. Gary et al. (1982) and Mehta and Sood (1992) concluded that when this effect is included, the flow characteristics substantially change compared to the constant case of viscosity. Recently, Kafoussius& Williams (1995) and Kafoussiaset al. [Kafoussias, Rees and Daskalakis (1998)] have investigated the effect of the temperature-dependent viscosity on the mixed convection flow past a vertical flat plate in the region near the leading edge using the local non-similarity method. In these studies, they concluded that when the viscosity of a fluid is sensitive to temperature variations, the effect of temperature-dependent viscosity has to be taken into consideration, otherwise considerable errors may occur in the characteristics of the heat transfer process. Hossain et al. [Hossain, Kabir and Rees (2002)] have investigated the natural convection of fluid with variable viscosity from a heated vertical wavy surface. Hossain& Munir(2000) investigated the mixed convection flow from a vertical flat plate for a temperature dependent viscosity. In the above studies [Hossain, Kabir and Rees (2002)] [Hossain & Munir (2000)] the viscosity of the fluid has been considered to be inversely proportional to a linear function of temperature.

According to the state of the art, variable viscosity and MHD effects on laminar boundary layer flow of the fluids along porous plate has not been studied yet. The variable physical property of viscosity may change significantly with temperature. In this study the presence of variable viscosity on MHD free convection boundary layer flow from a porous vertical plate of a steady two dimensional viscous incompressible fluid has been considered. Here we have investigated the variable viscosity on MHD natural convection flow from a porous vertical plate numerically. The results are obtained for different values of relevant physical parameters and shown in graphs as well as in tables.

The governing partial differential equations are reduced to locally non-similar partial differential

forms by adopting some appropriate transformations. The transformed boundary layer equations are solved numerically using implicit finite difference scheme together with the Keller Box technique[Keller (1978)]. Here, Our attention is focused on the evolution of the surface shear stress in terms of local skin friction and the rate of heat transfer in terms of local Nusselt number, velocity profiles as well as temperature profiles for selected values of parameters consisting of MHD M, variable viscosity γ, Prandtl number Pr.

Problem Description and Mathematical Model

To investigate MHD free convection flow from a porous plate with variable viscosity, the fluid is assumed to be a grey, emitting and absorbing but non scattering medium and the surface temperature of the porous vertical plate, T_w, is constant, where $T_w > T_\infty$. The considered physical configuration is shown in Fig. 1(A):

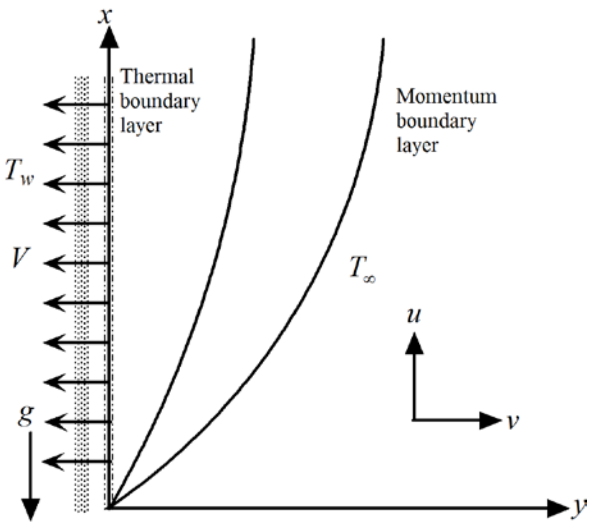

FIG. 1(A) THE COORDINATE SYSTEM AND THE PHYSICAL MODEL

The conservation equations for the flow characterized with steady, laminar and two dimensional boundary layer, under the usual Boussinesq approximation, the continuity, momentum and energy equations can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + \rho g\beta(T - T_\infty) - \sigma_0{\beta_0}^2 u \quad (2)$$

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\frac{\partial^2 T}{\partial y^2} \quad (3)$$

With the boundary conditions

$$\begin{aligned} &x = 0,\ y > 0,\ u = 0,\ T = T_\infty. \\ &y = 0,\ x > 0,\ u = 0,\ v = -V,\ T = T_W \\ &y \to \infty,\ x > 0,\ u = 0,\ T = T_\infty \end{aligned} \quad (4)$$

where ρ is the density, β_0 is the strength of magnetic field, σ_0 is the electrical conduction, k is the thermal conductivity, β is the coefficient of thermal expansion, ν is the reference kinematic viscosity $\nu = \mu/\rho$, μ is the viscosity of the fluid, C_p is the specific heat at constant pressure. The absolute viscosity μ is assumed to be vary with temperature according to a general functional form $\mu = \mu_f s(T)$, where μ_f is the absolute viscosity at the film temperature T_f and $s(T_f) = 1$. This form is chosen to allow definition of the stream function based on the absolute viscosity at the film temperature. For liquids, all transport properties vary with temperature. However, for many liquids like petroleum oils, glycerin, glycol, silicon fluids and some molten salt, the rate of variation of absolute viscosity with temperature is much more than that of the other properties. Under the above conditions, an analysis incorporating the above assumptions and describing the momentum and thermal transport within the flow field is more accurate than the usual assumption of constant properties evaluated at some reference temperature. It should be mentioned here that there are some fluids for which properties other than μ vary strongly with temperature. In particular, water and methyl alcohol exhibit strong variation of both μ and β. The analysis presented here is not applicable to these liquids since only the variation of the absolute viscosity is taken into consideration as a function of temperature. However, for the case of an isothermal surface (in an un-stratified ambient fluid), the variation of the absolute viscosity with temperature takes the form $\mu = \mu_f S(\theta)$, where θ is the dimensionless temperature in the boundary layer defined in equation (4), such that $S(1/2) = 1$. A wide variety of functional forms of $S(\theta)$ satisfying this requirement was investigated in the literature such as algebraic expressions, power series, exponential forms, etc. Carey & Mollendorf (1978), the simplest form of the absolute viscosity is used in this investigation as follows:

$$\mu = \mu_f\left[1 + \frac{1}{\mu_f}\left(\frac{d\mu}{dt}\right)_f (T - T_\infty)\right] \quad (5a)$$

This simple form amounts to a linear variation of the absolute viscosity with temperature, with the slope $d\mu/dT$, evaluated at film temperature. The assumed linear variation of viscosity with temperature gives rise to a new parameter γ defined by

$$\gamma = \frac{1}{\mu_f}\left(\frac{d\mu}{dt}\right)_f (T - T_\infty) \quad (5b)$$

Now introduce the following non-dimensional variables:

$$\eta=\frac{Vy}{\nu\xi}\,,\ \xi=V\left\{\frac{4x}{\nu^2 g\beta\Delta T}\right\}^{\frac{1}{4}},\ \psi=V^{-3}\nu^2 g\beta\Delta T\xi^3\left\{f+\frac{\xi}{4}\right\}$$

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}\,,\ \theta_w=\frac{T_w}{T_\infty} \tag{6}$$

Where, θ is the non-dimensional temperature function, θ_w is the surface temperature parameter.

Substituting (6) into Equations (1), (2) and (3) leads to the following non-dimensional equations

$$\left[1+\gamma\left(\theta-\frac{1}{2}\right)\right]f'''+\theta-2f'^2+3ff''+\xi f'' = \xi\left(f'\frac{\partial f'}{\partial\xi}f''\frac{\partial f'}{\partial\xi}\right)-\frac{\sigma_0\beta_0^2}{\rho}v^{-2}\xi^2 f' \tag{7}$$

$$\frac{1}{Pr}\frac{\partial}{\partial\eta}\left[\frac{\partial\theta}{\partial\eta}\right]+3f\theta'+\xi\,\theta'=\xi\left(f'\frac{\partial\theta}{\partial\xi}-\frac{\partial f}{\partial\xi}\theta'\right) \tag{8}$$

Where $Pr = \mu C_p/k$ is the Prandtl number and $M=\beta_0^2\sigma_0/v\rho$ is the hydromagnetic parameter. The boundary conditions (4) become

$$f=0,\ f'=0,\ \theta=1 \text{ at } \eta=0$$
$$f'=0,\ \theta=0 \text{ as } \eta\to\infty \tag{9}$$

The solution of equations (6), (8) enable us to calculate the nondimensional velocity components $\bar{u}$, $\bar{v}$ from the following expressions

$$\bar{u}=\frac{\nu^2}{Vg\beta(T_w-T_\infty)}u=\xi^2 f'(\xi,\eta)$$
$$\bar{v}=\frac{v}{V}=\xi^{-1}(3f+\xi-\eta f'+\xi\frac{\partial f}{\partial\xi}) \tag{10}$$

In practical applications, the physical quantities of principle interest are the shearing stress τ_w and the rate of heat transfer in terms of the skin-friction coefficients C_{fx} and Nusselt number Nu_x respectively, which can be written as

$$Nu_x=\frac{v}{V\Delta T}(q_c)_{\eta=0},\ C_{fx}=\frac{V}{g\beta\Delta T}(\tau)_{\eta=0} \tag{11}$$

$$\text{where }\ \tau_w=\mu\left(\frac{\partial u}{\partial y}\right)_{\eta=0} \text{ and } q_c=-k\left(\frac{\partial T}{\partial y}\right)_{\eta=0} \tag{12}$$

q_c is the conduction heat flux.

Using the Equations (6) and the boundary condition (9) into (11) and (12), we get

$$C_{fx}=\xi\left(1+\frac{\gamma}{2}\right)f''(x,0)$$
$$Nu_x=\xi^{-1}\theta'(x,0) \tag{13}$$

The values of the velocity and temperature distribution are calculated respectively from the following relations:

$$\bar{u}=\xi^2 f'(\xi,\eta),\quad \theta=\theta(x,y) \tag{14}$$

Numerical Procedure

Using implicit finite difference method with Keller-Box Scheme, the solution of the local non similar partial differential equation (7) to (8) subjected to the boundary condition (9) are obtained, which has been described in details by Cebeci and Bradshaw(1991).

The solution methodology of equations (7) and (8) with the boundary condition given in equation (9) for the entire ξ values based on Keller–box scheme is proposed here. The scheme specifically incorporated a nodal distribution favoring the vicinity of the plate, enabling accuracy to be maintained in this region of steep gradient. In detail, equations (7) and (8) are solved as a set of five simultaneous equations.

$$\left[1+\gamma\left(\theta-\frac{1}{2}\right)f''\right]'+3ff''-2(f')^2+\theta-\xi f''-Mf'\xi^2 = \xi\left(f'\frac{\partial f'}{\partial\xi}-\frac{\partial f}{\partial\xi}f''\right) \tag{15}$$

and

$$\frac{1}{\Pr}\frac{\partial}{\partial\eta}\left[\frac{\partial\theta}{\partial\eta}\right]+3f\theta'+\xi\theta'=\xi\left(f'\frac{\partial\theta}{\partial\xi}-\frac{\partial f}{\partial\xi}\theta'\right) \tag{16}$$

To apply the aforementioned method, we first convert Equations (15)-(16) into the following system of first order equations with dependent variables $u(\xi,\eta)$, $v(\xi,\eta)$, $p(\xi,\eta)$ and $g(\xi,\eta)$ as

$$f'=u,\ u'=v,\ g=\theta, \text{ and } \theta'=p \tag{17}$$

$$\left[1+p_5\left(g-\frac{1}{2}\right)v\right]'+p_1fv-p_2u^2+g-\xi v-p_4u\xi^2 = \xi\left(u\frac{\partial u}{\partial\xi}-\frac{\partial f}{\partial\xi}v\right) \tag{18}$$

$$\frac{1}{\Pr}[p']+\xi p+p_1fp=\xi\left(u\frac{\partial g}{\partial\xi}-p\frac{\partial f}{\partial\xi}\right) \tag{19}$$

where

$$p_1=3,\ p_2=2,\ p_4=M \text{ and } p_5=\gamma \tag{20}$$

The corresponding boundary conditions are

$$f(\xi,0)=0,\ u(\xi,0)=0 \text{ and } g(\xi,0)=0$$
$$u(\xi,\infty)=0,\ g(\xi,\infty)=0 \tag{21}$$

We now consider the net rectangle on the (ξ,η) plane and denote the net point by

$$\eta_0=0,\ \eta_j=\eta_{j-1}+h_j,\quad j=1,2,\ldots J$$
$$\xi^0=0,\quad \xi^n=\xi^{n-1}+k_n,\quad n=1,2,\ldots N$$

Here *'n'* and *'j'* are just sequence of numbers on the (ξ,η) plane, k_n and h_j are the variable mesh widths.

We approximate the quantities (f, u, v, p) at the points

(ξ^n,η_j) of the net by $(f_j^n, u_j^n, v_j^n, p_j^n)$ which we call net function.

$$\eta_{j-1/2}=\frac{1}{2}(\eta_j-\eta_{j-1}) \tag{22}$$
$$\xi^{n-1/2}=\frac{1}{2}(\xi^n+\xi^{n-1})$$
$$g_j^{n-1/2}=\frac{1}{2}(g_j^n+g_j^{n-1})$$
$$g_{j-1/2}^n=\frac{1}{2}(g_j^n+g_{j-1}^n)$$

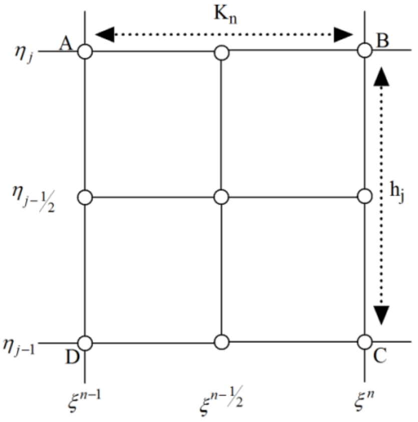

FIG. 1(B) NET RECTANGLE FOR DIFFERENCE APPROXIMATIONS FOR THE BOX SCHEME

Now we write the difference equations that are to approximate Equations (17)-(19) by considering one mesh rectangle for the mid-point $(\xi^n, n_{j-1/2})$ to obtain

$$\frac{f_j^n-f_{j-1}^n}{h_j}=u_{j-1/2}^n \tag{23}$$

$$\frac{u_j^n-u_{j-1}^n}{h_j}=v_{j-1/2}^n \tag{24}$$

$$\frac{g_j^n-g_{j-1}^n}{h_j}=p_{j-1/2}^n \tag{25}$$

Similarly, Equations (18)–(19) are approximate by centering about the midpoint $(\xi^{n-1/2}, n_{j-1/2})$. Centering the Equations (22) about the point $(\xi^{n-1/2}, n)$ without specifying η to obtain the algebraic equations. The difference approximation to Equations (18)-(19) becomes

$$\frac{h_j^{-1}}{2}\left[\begin{array}{l}\{1+p_5(g-0.5)v\}_j^n-\{1+p_5(g-0.5)v\}_{j-1}^n\\ +\{1+p_5(g-0.5)v\}_j^{n-1}-\{1+p_5(g-0.5)v\}_{j-1}^{n-1}\end{array}\right]$$
$$+\{(p_1)_{j-1/2}^n+\alpha_n\}(fv)_{j-1/2}^n-\{(p_2)_{j-1/2}^n+\alpha_n\}(u^2)_{j-1/2}^n$$
$$+g_{j-1/2}^n-(p_4u\xi^2)_{j-\frac{1}{2}}^{n-\frac{1}{2}}-(\xi v)_{j-1/2}^n+\alpha_n\{f_{j-1/2}^n v_{j-1/2}^{n-1}$$
$$-v_{j-1/2}^n f_{j-1/2}^{n-1}\}=R_{j-1/2}^{n-1}$$

where

$$L_{j-\frac{1}{2}}^{n-1}=h_j^{-1}\left[\{1+p_5(g-0.5)\}_j^{n-1}-\{1+p_5(g-0.5)\}_{j-1}^n\right]$$
$$+(p_1)_{j-\frac{1}{2}}^{n-1}(fv)_{j-\frac{1}{2}}^{n-1}-(p_2)_{j-\frac{1}{2}}^{n-1}(u^2)_{j-\frac{1}{2}}^{n-1}+g_{j-\frac{1}{2}}^{n-1}$$
$$-(\xi p)_{j-\frac{1}{2}1}^{n-1}+h_j^{-1}\left(v_j^{n-1}-v_{j-1}^{n-1}\right)-(p_4u\xi^2)_{j-1}^{n-1}$$

and

$$R_{j-\frac{1}{2}}^{n-1}=-L_{j-\frac{1}{2}}^{n-1}+\alpha_n\left\{-(u^2)_{j-\frac{1}{2}}^{n-1}+(fv)_{j-\frac{1}{2}}^{n-1}\right\}$$

$$\Rightarrow\frac{1}{\Pr}[h_j^{-1}(p_j^n-p_{j-1}^n)+h_j^{-1}\{\{p_3p(1+\Delta g)^3\}_j^n$$
$$-\{p_3p(1+\Delta g)^3\}_{j-1}^n\}]+\xi_{j-1/2}^n p_{j-1/2}^n+(p_1)_{j-1/2}^n(fp)_{j-1/2}^n$$
$$=-M_{j-1/2}^{n-1}+\alpha n[-(ug)_{j-1/2}^{n-1}+(fp)_{j-1/2}^{n-1}]$$
$$+\alpha_n[(ug)_{j-1/2}^n-(fp)_{j-1/2}^n-u_{j-1/2}^n g_{j-1/2}^{n-1}+u_{j-1/2}^{n-1}g_{j-1/2}^n\}$$
$$+p_{j-1/2}^n f_{j-1/2}^{n-1}-p_{j-1/2}^{n-1}f_{j-1/2}^n]$$

$$\Rightarrow\frac{1}{\Pr}[h_j^{-1}(p_j^n-p_{j-1}^n)]+\xi_{j-1/2}^n p_{j-1/2}^n+(p_4\xi^2)_{j-1/2}^n g_{j-1/2}^n$$
$$+\{(p_1)_{j-1/2}^n+\alpha_n\}(fp)_{j-1/2}^n$$
$$-\alpha_n[\{(ug)_{j-1/2}^n-(ug)_{j-1/2}^{n-1}-u_{j-1/2}^n g_{j-1/2}^{n-1}+u_{j-1/2}^{n-1}g_{j-1/2}^n\}$$
$$+p_{j-1/2}^n f_{j-1/2}^{n-1}-p_{j-1/2}^{n-1}f_{j-1/2}^n\}]$$
$$=T_{j-1/2}^{n-1}$$

where

$$M_{j-1/2}^{n-1}=\frac{1}{\Pr}[h_j^{-1}(p_j^{n-1}-p_{j-1}^{n-1})]$$
$$-[\xi_{j-1/2}^{n-1}p_{j-1/2}^{n-1}+(p_1)_{j-1/2}^{n-1}(fp)_{j-1/2}^{n-1}]$$

$$T_{j-1/2}^{n-1}=-M_{j-1/2}^{n-1}+\alpha_n[(fp)_{j-1/2}^{n-1}-(ug)_{j-1/2}^{n-1}]$$

The corresponding boundary conditions (21) become

$$f_0^n=0,\ u_0^n=0,\ g_0^n=1$$
$$u_J^n=0,\ g_J^n=0$$

which just express the requirement for the boundary conditions to remain during the iteration process. Now the momentum and energy equations will be converted into system of linear Equations and together with the boundary conditions can be written in matrix or vector form, where the coefficient matrix has a block tri-diagonal structure. The whole procedure, namely reduction to first order followed by central difference approximations, Newton's quasi-linearization method and the block Thomas algorithm, is well known as the Keller-box method.

Results and Discussion

To investigate the MHD natural convection flow from a porous vertical plate with variable viscosity, numerical values of local rate of heat transfer was calculated in terms of Nusselt number Nu_x for the surface of the porous vertical plate from lower stagnation point to upper stagnation point. Simulation was performed for different values of the aforementioned parameters and those are shown in tabular form in Table 1 and graphically in Figure 5-7. The effect for different values viscosity γ on local skin friction coefficient C_{fx} and the local Nusselt number Nu_x, as well as velocity and temperature profiles are displayed in Figure 2 to 7.

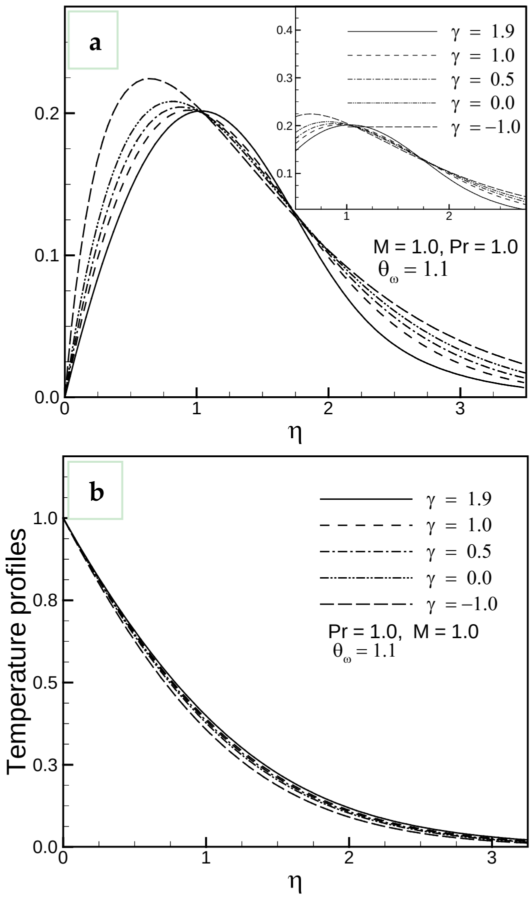

FIG. 2(a) VELOCITY AND (b) TEMPERATURE PROFILES FOR DIFFERENT VALUES OF VISCOSITY PARAMETER Γ WITH OTHERS FIXED PARAMETERS.

Keeping constant the Prandtl number Pr = 1.0, magnetohydrodynamic parameter M = 1.0 surface temperature parameter θ_w = 1.1 and figures 2(a)-2(b) display results for the velocity and temperature profiles for different values of viscosity parameter γ= -1.0, 0.0, 0.5, 1.0, 1.9. Figures 2(a)-2(b) shows that as the viscosity parameter γ increases, the velocity profiles decreases and the temperature profiles increase. The fact is that due to the viscosity it takes away the warm fluid and thereby decreasing the maximum velocity with a decreasing intensity of the natural convection rate as depicted be figure 2 (a). The velocity is zero at the boundary wall then the velocity starts increasing to the peak value while η increases from 1 to 1.8 and then starts decreasing after η = 1.8 and so on. Finally the velocity approaches to zero (the asymptotic value).

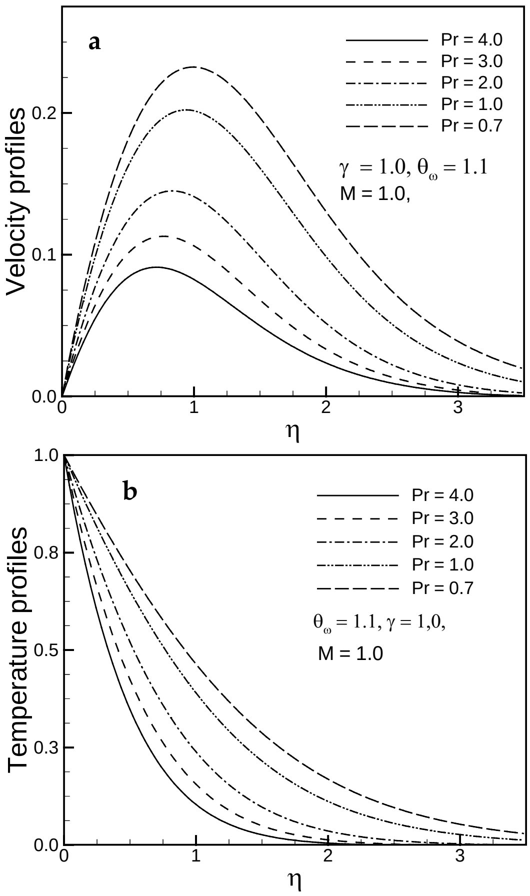

FIG. 3(a) VELOCITY AND (b) TEMPERATURE PROFILES FOR DIFFERENT VALUES OF PRANDTL NUMBER PR WITH OTHERS FIXED PARAMETERS.

However, in figures 3(a)-3(b) it has been shown that keeping constant θ_w = 1.0, M = 1.0 and γ= 0.5 while the Prandtl number Pr (= 0.7, 1.0, 2.0, 3.0 and 4.0)increases both the velocity and temperature profiles decrease.

This is due to the fact that as *Pr* increases, viscosity increases so the velocity decreases.

Figure 4(a) shows the velocity profiles for varying *M* from 0 to 30 with Prandtl number *Pr* = 1.0, viscosity parameter γ = 1.0 and surface temperature parameter θ_w = 1.1. It is shown in figure 4(a) that as the MHD parameter increases the velocity profiles decrease because electrically conducting fluid affects the flow so the velocity decreases. It is also observed from figure 4(a) that the changes of velocity profiles in the η direction reveal the typical velocity profile for natural convection boundary layer flow. With an increment ofη, the velocity profile shows an asymptotic behaviour. The maximum values of velocity are recorded to be 0.11934 and 0.14053 at η = 0.88811 and 0.16893, 0.18645, 0.20630 at η= 0.94233 for *M* =.30.0, 20.0,10.0, 5.0 and 0.0 respectively. The velocity is 0.20630 at η= 0.94233 for *M* = 0.0. Here, it is observed that at η= 0.88811, the velocity decreases by 73.40% as the MHD parameter *M* changes from 0 to 30.0.

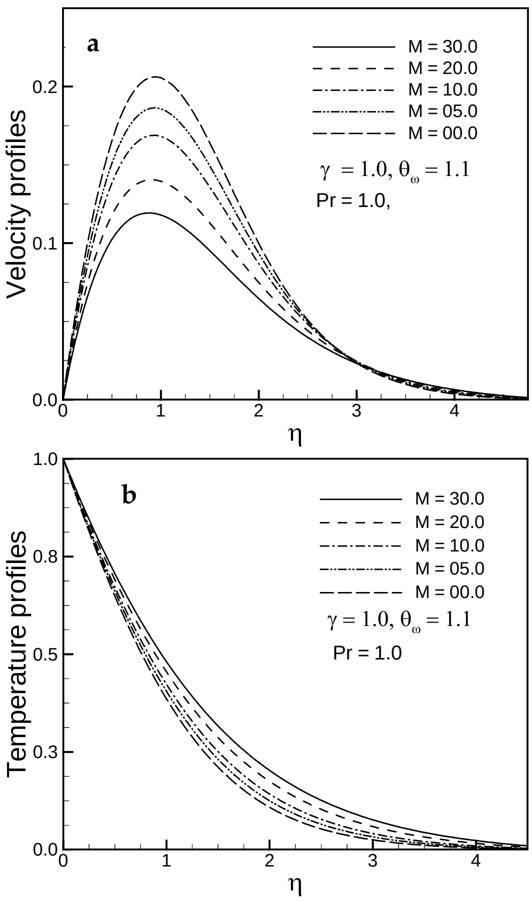

FIG. 4(a) VELOCITY AND (b) TEMPERATURE PROFILES FOR DIFFERENT VALUES OF MHD PARAMETER M WITH OTHERS FIXED PARAMETERS.

Figure 4(b) shows results for the temperature profiles, for different values of MHD parameter *M* while Prandtl number *Pr* = 1.0, viscosity parameter γ = 1.0 and surface temperature parameter θ_w = 1.1. From figure 4(b) it infers that as the MHD parameter *M* increases, the temperature profiles increase. It is observed that the temperature profile decreases gradually along η direction from 1.0 to 0 with an asymptotic behaviour. For *M* = 30.0, 20.0, 10.0, 5.0, 0.0 the temperature profile raise to 0.52579, 0.50164, 0.47181, 0.45488 and 0.43689 at η= 0.88811 then it starts decreasing. And for other values, thoseare gradually increasing.

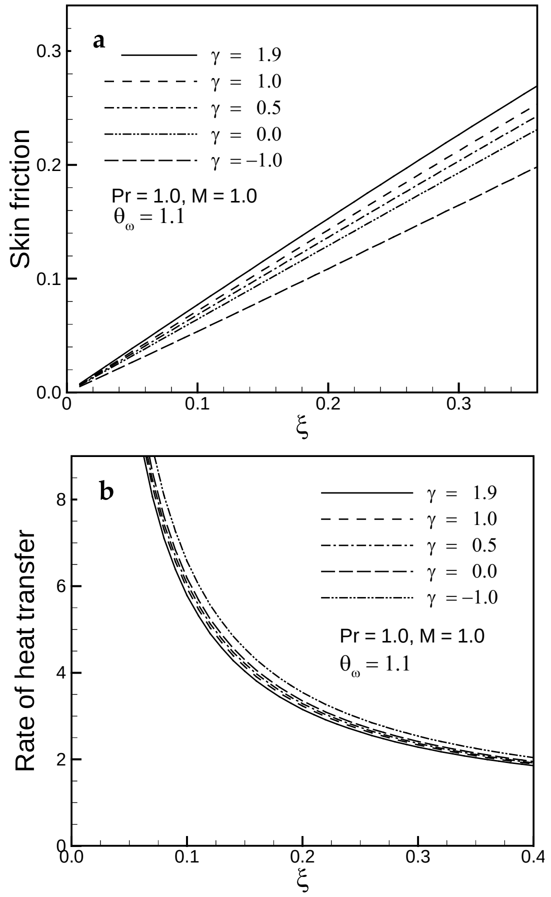

FIG. 5 (a) SKIN FRICTION AND (b) RATE OF HEAT TRANSFER FOR DIFFERENT VALUES OF VISCOSITY PARAMETER Γ WITH OTHERS FIXED PARAMETERS.

Figure 5 (a) shows that skin friction coefficient C_{fx} increases with viscosity parameter γconsidering Prandtl number *Pr* = 1.0, surface temperature parameter θ_w = 1.1 and MHD parameter *M* = 1.0. It is observed from figure 5(a) that the skin friction increases gradually from zero at lower stagnation point along the ξ direction. Figure 5(b) reveals that the rate of heat transfer decreases along the ξ direction for γ= -1.0, 0.0, 0.5, 1.0 and 1.9. A hot fluid layer is created adjacent to the interface of the wall due to the viscosity mechanism and ultimately the resultant temperature

of the fluid exceeds the surface temperature. Accordingly, the heat transfer rate from the surface decreases as shown in Fig. 5(b).

The variation of the local skin friction coefficient C_{fx} and local rate of heat transfer Nu_x for different values of Prandtl number Pr while $\theta_w = 1.0$, $M = 1.0$ and $\gamma = 1.0$ are shown in Figures 6(a)-6(b). It can be observed from these figures that as the Prandtl number Pr increases, the skin friction coefficient decreases and the rate of heat transfer increases.

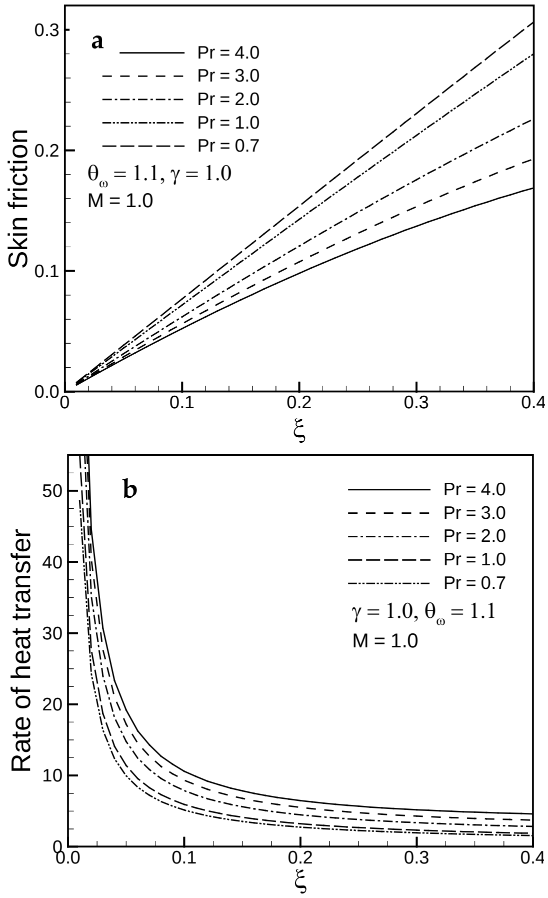

FIG. 6(a) SKIN FRICTION AND (b) RATE OF HEAT TRANSFER FOR DIFFERENT VALUES OF PRANDTL NUMBER PR WITH OTHERS FIXED PARAMETERS .

Figures 7(a)-7(b) show that skin friction coefficient C_{fx} and heat transfer coefficient Nu_x decrease for increasing values of MHD parameter M while viscosity parameter $\gamma = 1.0$, Prandtl number $Pr = 1.0$, and surface temperature parameter $\theta_w = 1.1$. The values of skin friction coefficient C_{fx} and Nusselt number Nu_x are recorded to be 0.25646, 0.30238, 0.39508, 0.50394. 0.87147 and 1.00382, 1.00463, 1.00414, 1.00463, 1.04285 for M = 30.0, 20.0, 10.0.5.0, 0.0 respectively which occur at the same point $\xi = 1.5$. Here, it observed that at $\xi = 1.5$, the skin friction decreases by 70.57% and Nusselt number Nu_x decreases by 3.74% as the MHD parameter M changes from 0.0 to 30.0. It is observed from figure 7(a) that the skin friction increases gradually from zero at lower stagnation point along the ξ direction. From Figure 7(b) it is revealed that the rate of heat transfer decreases along the ξ direction.

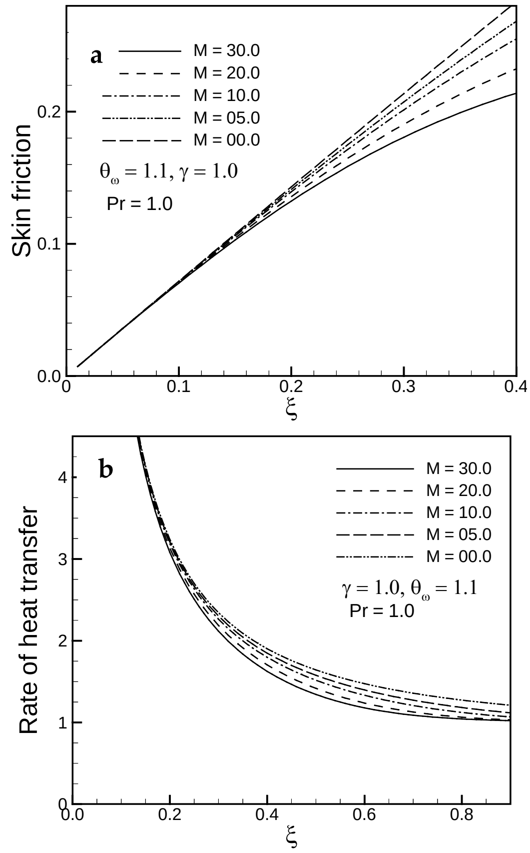

FIG. 7(a) SKIN FRICTION AND (b) RATE OF HEAT TRANSFER FOR DIFFERENT VALUES OFMHD PARAMETER Q WITH OTHERS FIXED PARAMETERS

Numerical values of rate of heat transfer Nu_x and skin friction coefficient C_{fx} are calculated from Equations (13) from the surface of the vertical porous plate. Numerical values of C_{fx} and Nu_x are shown in Table 1.

TABLE 1 SKIN FRICTION COEFFICIENT AND RATE OF HEAT TRANSFER AGAINST ξ FOR DIFFERENT VALUES OF MHD PARAMETER M WITH OTHER CONTROLLING PARAMETERS $P_R = 1.0$, $\theta_W = 1.1$. AND $\gamma = 1.0$.

ξ	M = 30.0		M = 20.0	
	C_{fx}	Nu_x	C_{fx}	Nu_x
0.01	0.00720	55.24526	0.00720	55.25031
0.05	0.03579	11.37380	0.03586	11.38889
0.10	0.07033	5.86455	0.07081	5.89219
0.50	0.23492	1.34271	0.26201	1.41711
1.00	0.25731	1.01311	0.30335	1.01638
1.50	0.25646	1.00382	0.30238	1.00463
ξ	M = 10.0		M = 0.0	
	C_{fx}	Nu_x	C_{fx}	Nu_x
0.01	0.00720	55.25537	0.00720	55.26042
0.05	0.03593	11.40399	0.03600	11.41913
0.10	0.07130	5.92000	0.07180	5.94800

0.50	0.29917	1.51681	0.35101	1.64306
1.00	0.39248	1.03533	0.65607	1.16318
1.50	0.39508	1.00414	0.87147	1.04285

In the above table, the values of skin friction coefficient C_{fx} and Nusselt number Nu_x are recorded to be 0.25646, 0.30238, 0.39508, 0.87147 and 1.00382, 1.00463, 1.00414, and 1.04285 for M = 30.0, 20.0, 10.0, 5.0, 0.0 and respectively which occur at the same point ξ = 1.5. Here, it is observed that at ξ = 1.5, the skin friction decreases by 70.57% and Nusselt number Nu_x decreases by 3.74% as the MHD parameter M changes from 0.0 to 30.

Comparison of the Results

In order to verify the accuracy of the present work, the values of Nusselt number and skin friction for R_d = 0.05, Pr = 1.0, γ = 0, M = 0 and various surface temperature parameter θ_w = 1.1, θ_w = 2.5 at different position of ξ are compared with Hossain et al. (Hossain, Alim, and Rees) as presented in Table 2. The results are found to be in excellent agreement.

TABLE 2 COMPARISON OF THE PRESENT TABLE WITH HOSSAIN ET AL(Hossain, Alim, and Rees)

ξ	θw = 1.1			
	Hossain		Present	
	C_{fx}	Nu_x	C_{fx}	Nu_x
0.1	0.0655	6.4627	0.06535	6.48306
0.2	0.1316	3.4928	0.13138	3.50282
0.4	0.2647	2.0229	0.26408	2.03018
0.6	0.3963	1.5439	0.39519	1.55522
0.8	0.5235	1.3247	0.52166	1.32959
1.0	0.6429	1.1995	0.64024	1.20347
1.5	0.8874	1.0574	0.88192	1.06109
ξ	θw = 2.5			
	Hossain		Present	
	C_{fx}	Nu_x	C_{fx}	Nu_x
0.1	0.0709	8.0844	0.07078	8.10360
0.2	0.1433	4.2858	0.14313	4.29682
0.4	0.2917	2.4003	0.29120	2.40669
0.6	0.4423	1.7863	0.44145	1.78912
0.8	0.5922	1.4860	0.59080	1.48991
1.0	0.7379	1.1098	0.73590	1.31822
1.5	1.0613	1.1098	1.05693	1.11262

Conclusion

For different values of relevant physical parameters including the viscosity parameter γ, MHD natural convection with variable viscosity flow from a porous vertical plate has been investigated and the following conclusions may be drawn:

- Significant effects of MHD parameter M and viscosity parameter γ on velocity and temperature profiles as well as on skin friction coefficient$C_{fx}$and the rate of heat transfer Nu_x have been found in this investigation but the effect of MHD parameter M and viscosity parameter γ on rate of heat transfer is more significant. An increase in the values of viscosity parameter γ leads to the decrement of velocity and the increment of temperature profiles and the local skin friction coefficient C_{fx} increase and the local rate of heat transfer Nu_x decreasing at different position of ξ for Pr =1.0.
- For increasing values of Prandtl number Pr leads to decrease of the velocity profile, the temperature profile and the local skin friction coefficient C_{fx} but increment of the local rate of heat transfer Nu_x.
- An increase in the values of M leads to increase of the temperature profiles and the velocity profiles, the local skin friction coefficient C_{fx} and the decrease of local rate of heat transfer Nu_x.

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